Enhancing Statistics Learning through Automated STACK Testing

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TTK UAS Research Problem

The challenge arises due to the limited contact hours available to lecturers for an in-depth study of classical theoretical distributions of random variables. This necessitates the exploration of alternative methods to enable students to thoroughly grasp and reinforce the curriculum of the statistics subject, while also monitoring their own knowledge acquisition.

The study is conducted within the framework of a statistics course at TTK UAS. This article details the development and implementation of an automated testing system, focusing on one of the test topics: discrete binomial distribution.

Part 1

```
/* VARIABLES */
p:rand_with_step(0.3,0.9,0.05)
n:rand_with_step(4,7,1)
x_n: makelist(i,i,0,n)
fun(x):=binomial(n,x)*p^x*(1-p)^(n-x)
P_x: makelist(decimalplaces(fun(x),3),x,0,n)
P_x0: makelist(fun(x),x,0,n)
 cumsum(L):=makelist(sum(L[i],i,1,n),n,1,length(L))
F_x: decimalplaces(cumsum(P_x0),3)
```

Random group: p, n, x_n, fun(x), P_x, P_x0, F_x

Part 2.1

```
/* VARIABLES*/
 Points_Fx_list: makelist ([x_n[i], F_x[i]], i, 1, n+1)
 Points_Px_list: makelist ([x_n[i], P_x[i]], i, 1, n+1)
```

Part 2.2

```
/*The Cumulative Distribution Function (cdf) plot*/
/*cdf function = Piecewise defined function*/
 q(x,x0,y0):=if (x>=x0 and x<x0+1) then y0
 cdf:makelist(q(x,k-1,F_x[k]),k,1,n+1)

 /*ANSWERS*/

plot1: plot(data_list2,[x,0,n],[style,[lines,3,2]],[ytics,0,0.25,1],[size,250,250],
                    [xlabel,"X"],[ylabel,"F(x)"],[plottags,false])
fun_wrong1(x):=binomial(n-1,x)*(p-0.1)^x*(1-p)^(n-x)
plot2: plot(cdf,[x,0,n+1],[style,[lines,3,2]],[xlabel,"X"],[ylabel,"F(x)"],
                    [ytics,0,0.25,1],[size,250,250],[plottags,false])
plot3: plot(fun_wrong1(x),[x,0,n],[style,[lines,3,2]],[xlabel,"X"],[ylabel,"F(x)"],
                    [size,250,250],[plottags,false])
cdf_wrong:makelist(q(x,k,F_x[k]),k,1,n)
plot4: plot(cdf_wrong,[x,0,n],[style,[lines,3,2]],[xlabel,"X"],[ylabel,"F(x)"],
                    [box,false],[ytics,0,0.25,1],[size,250,250],[plottags,false])

ta:[[1,false,plot1],[2,true,plot2],[3,false,plot3],[4,false,plot4]];
ta:random_permutation(ta);
```

Part 3.1

```
/* mean*/
   Ex: 0
   for x:0 thru n step 1 do Ex:fun(x)*x+Ex
   Ex:decimalplaces(Ex,2)

 /*variance and standard deviation*/
   Ex_2: 0;
   for x:0 thru n step 1 do Ex_2:fun(x)*x*x+Ex_2
   Dx: Ex_2-Ex^2
   Dx:decimalplaces(Dx,2)
   si: decimalplaces(sqrt(Dx),2)
```

Part 3.2

```
  /*mode*/
 Mo: sublist_indices(P_x, lambda([x1],x1 = lmax(P_x)))

/* Median, Quartilies*/
 Me: sublist_indices(F_x, lambda([x1],x1>=0.5))
 Q_1:sublist_indices(F_x, lambda([x1],x1>=0.25))
 Q_3:sublist_indices(F_x, lambda([x1],x1>=0.75))
```

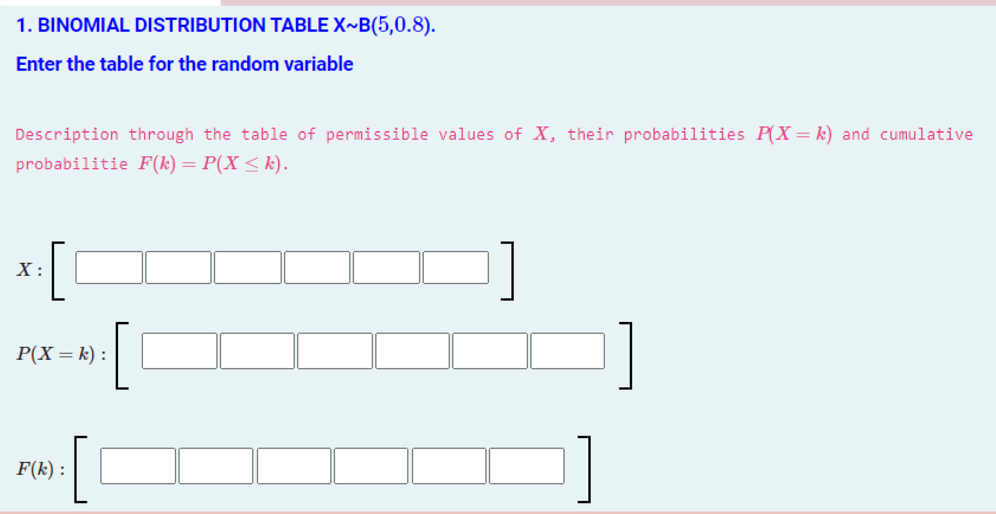
TASK

A target is shot at independently 5 times. The probability of hitting the target witch one shot is 0.8.
Random variable X - the total number of hits out of 4 shots.

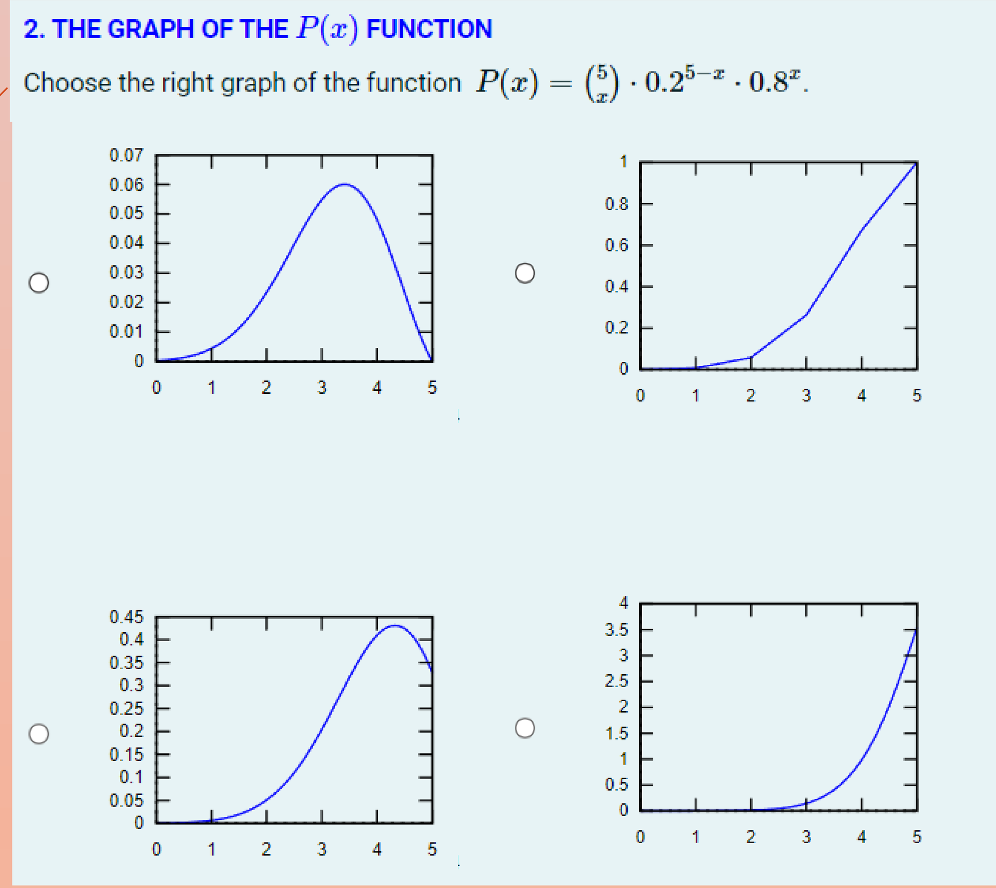
Describe the random variable X.

Results

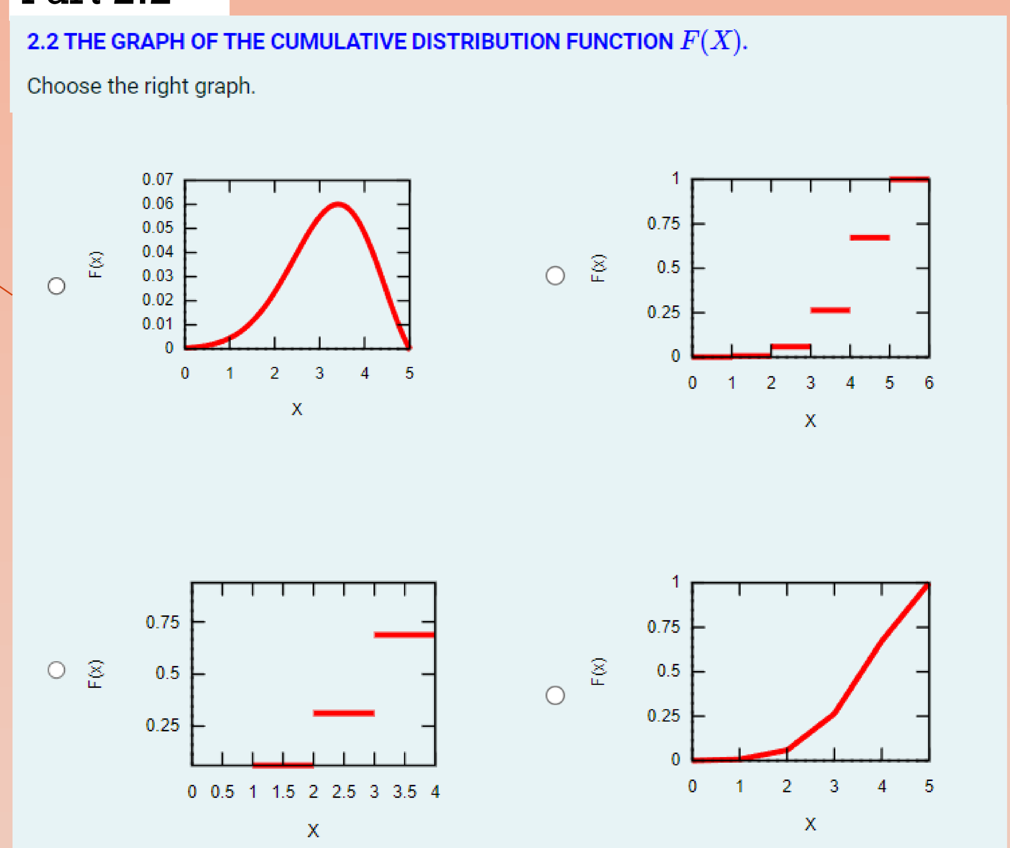
Part 1



Part 2.1



Part 2.2



Part 3.1

3.1 DESCRIPTIVE STATISTICS: NUMERICAL MEASURES

MEAN: $EX =$

VARIANCE: $DX =$

STANDARD DEVIATION: $\sigma =$

Part 3.2

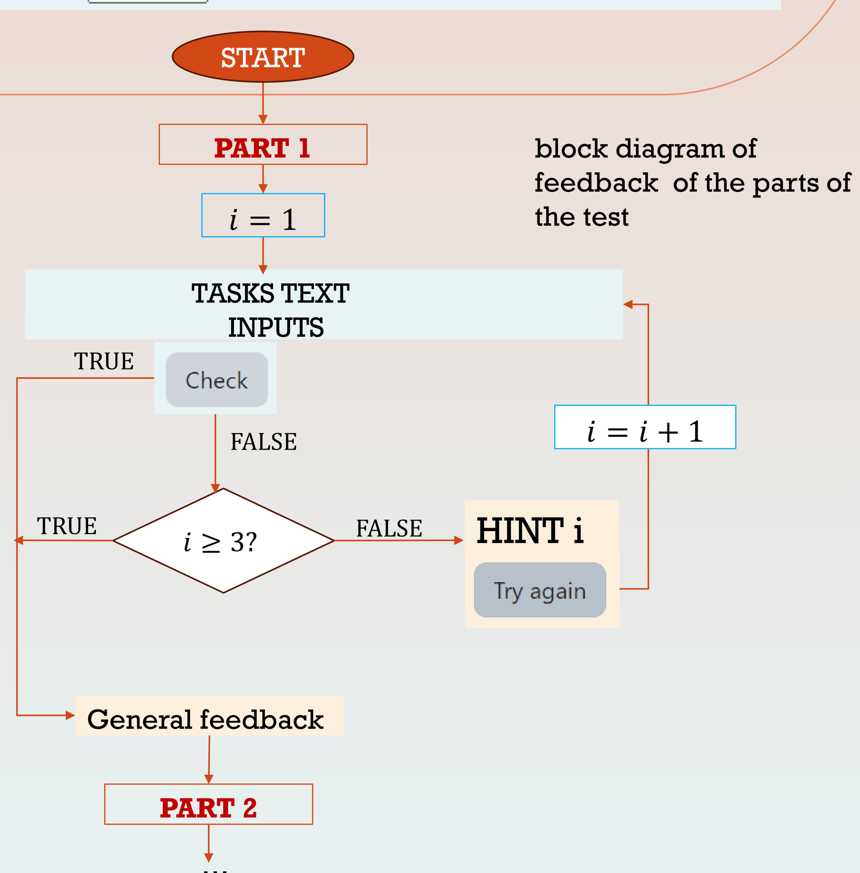
3.2 DESCRIPTIVE STATISTICS: NUMERICAL MEASURES

MODE: $M_o =$

MEDIAN: $M_e =$

LOWER QVARTILE: $Q_1 =$

UPPER QVARTILE: $Q_3 =$



block diagram of feedback of the parts of the test

Methodology

Binomial distribution is a common discrete distribution used in statistics. Let X be a discrete random variable, being the number of successes occurring in n independent trials of an experiment. If X is to be described by the binomial model, the probability of exactly k successes in n trials is given by

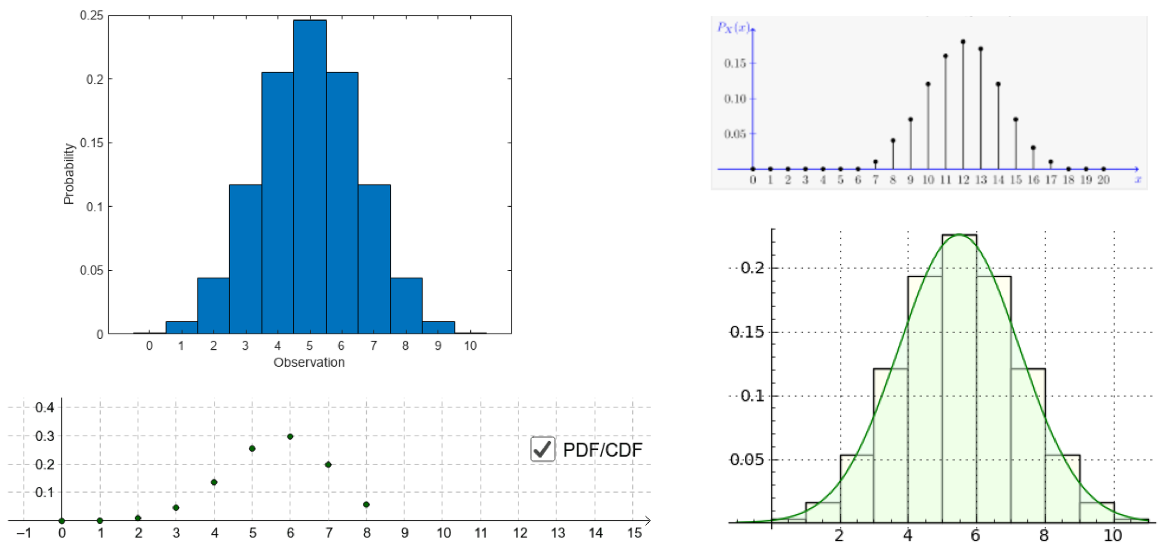
$$P(X=k)=C_n^k p^k q^{n-k}$$

Here there are k successes (each with probability p), $n-k$ failures (each with probability q) and $C_n^k=\frac{n!}{k!(n-k)!}$ the number of ways of placing the k successes among the n trials.

The task for describing the distribution of a random discrete variable is constructed according to the following scheme:

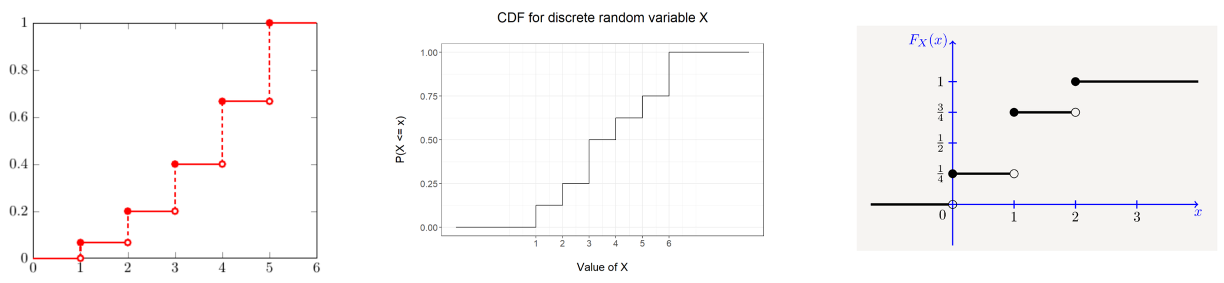
1) Description through a table of permissible values of X, their probabilities $P(X=k)=p_k$ and cumulative probabilities $F(k)=P(X\le k)$.
2) Graphical presentation of the Probability Density Function (PDF) and Cumulative Distribution Function (CDF).
3) Calculation of numerical measures.

PDF plots is a bar graph. The following graphs are often used in programs: a set of graphs, a set of lines, and a distribution curve.



CDF plots shows the empirical cumulative distribution function of the data. CDF plot for discrete distribution is a piecewise defined function:

$$F(x)=\begin{cases}0 & ,\text{for } x<0\\ P(X=0), & \text{for } 0\le x<1\\ P(X\le 1), & \text{for } 1\le x<2\\ \dots & \\ 1 & ,\text{for } x\ge n\end{cases}$$



3) Numerical measures and calculation formulas:

Mean: $EX=\sum x_k p_k$

Dispersion: $DX=E(X^2)-(EX)^2$

Standard Deviation: $\sigma=\sqrt{DX}\sigma$

Mode: *The mode is the value that appears with the highest frequency.*

Median: *The 1st value from the left for which F(x)≥0.5*

Lower Quartile: *The 1st value from the left for which F(x)≥0.25*

Upper Quartile: *The 1st value from the left for which F(x)≥0.75*

The graphs were generated using the plot() command.

Solved Problems

- Plotting piecewise defined functions.
- Incorporating graphs into multiple-choice questions to display images instead of text.
- Randomly shuffling schedules during each iteration.
- Identifying the correct answer corresponding to the graph in Sections 2.1 and 2.2.

Unsolved Problems

- Plot style for functions list and points list for part 2.2.
- Bar plot for part 2.1.

Conclusion

Implemented

We broke up and simplified large objects (such as splitting one task into parts or dividing a table into rows), which facilitated the setup of general feedback and hints in a more targeted and effective manner.

In the future, we intend to investigate the feasibility of automatically generating or modifying the task text and providing individual feedback using a potential response tree (PRT).

We will pilot the question with students next year and refine it based on their feedback.

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