

# INPUT DEMAND AND THE SHORT- AND LONG-RUN EMPLOYMENT THRESHOLDS

## An Empirical Analysis for the German Manufacturing Sector

### Abstract

The concept of the “employment threshold” plays an important role in the public discussion of unemployment. The employment threshold is defined as that growth rate of output necessary to keep employment constant despite the continuous rise in productivity. It is related to the Okun coefficient which describes the relationship between the changes in output and unemployment. Many contributions to this debate give the impression that the employment threshold is more or less a structural characteristic independent of economic variables. In this paper we derive short- and long-run employment thresholds from an input demand system and show empirically that they depend on factor prices and capital accumulation. Higher wage rates raise the employment threshold and reduce the probability that a positive output shock will increase employment.

Keywords: Okun’s Law, Employment Threshold, Labour, Productivity, Factor Demand

JEL Classification: D24, E24, L60

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## 1 Introduction

During the last two decades unemployment has been one of the most serious economic problems in many European countries. A key issue in the long lasting debate about this problem is the role of wages in determining the level of employment. Whereas some proponents of demand side policies argue that higher wages induce an increase in output demand and hence in employment, the opponents stress that wage increases will lead to a substitution of labour by capital and to higher output prices which will at least partly offset the initial expansionary effect of higher wages.

In the following study we address some of these problems within a unifying framework of cost minimisation and price setting behaviour which is modelled as a variable mark-up over marginal costs. Since we are using a restricted cost function with capital as a quasi-fixed input we are able to distinguish between the short-run and long-run effects of input prices on labour demand. Within this framework we derive short-run and long-run “employment thresholds” which play an important role at least in the German debate about the employment problem. The employment threshold is defined as that output growth rate necessary for a constant employment level. In contrast to many previous studies which treat the employment threshold as an exogenous parameter, we explicitly relate this variable to factor price changes, capital accumulation and time varying technical progress.

This approach has also some relevance for Okun’s Law which is used by many applied macro-economists as the link between changes in output and unemployment. Based on our empirical findings we argue that neglecting the effect of factor price changes implies a serious misspecification so that one cannot hope to get a stable relationship between the two variables.

The paper is organised as follows: Section 2 presents the theoretical foundations for the derivation of the employment thresholds from a cost minimisation approach. Section 3 explains the econometric specification. In Section 4 the empirical results are presented and discussed. Based on the most important results some conclusions are drawn in Section 5.

## 2 Theoretical foundations

### 2.1 The model

In the following we assume that a representative firm uses the inputs labour  $L$ , intermediate products  $M$  and capital  $K$  to produce the output  $Y$ . While labour (measured as labour volume which denotes the product of the number of employed persons and the amount of time they work) and intermediate products (material, energy) are variable inputs which can be changed without adjustment costs, due to convex adjustment costs or external restrictions capital is treated as a quasi-fixed input. Quasi-fixed inputs are predetermined in the short-run, but can varied in the long-run (typically in some variant of a flexible accelerator).

If firms minimise their short-run production costs at given input prices and given levels of output and capital there exists under weak assumptions with regard to production technology (see e.g. McFadden 1978 or Chambers 1988) a variable cost function:

$$CV = CV(q_m, q_l, K, Y, t) \quad (1)$$

$CV$  indicates the minimum variable cost of producing the output  $Y$  at given prices  $q_m$  and  $q_l$  of the variable production factors intermediate inputs  $M$  and labour  $L$  and at the given level of capital stock  $K$ . The time index  $t$  represents the state of the technology. In order to be able to represent all economically relevant information of the underlying technology the variable cost function must meet certain regularity conditions (Lau 1978, McFadden 1978):  $CV$  must be decreasing in  $K$  and increasing in  $q_l, q_m$  and  $Y$ . Moreover,  $CV$  has to be convex in  $K$  and concave and linearly homogenous in  $q_l$  and  $q_m$ .

The short-run demand functions for the variable factors are derived via Shepard's Lemma:

$$L(q_m, q_l, K, Y, t) = \frac{\partial CV}{\partial q_l} \quad (2)$$

$$M(q_m, q_l, K, Y, t) = \frac{\partial CV}{\partial q_m} \quad (3)$$

The model is supplemented by taking a supply relation into consideration:

$$P = \frac{\partial CV}{\partial Y} * \Theta, \quad (4)$$

where  $P$  represents the output price and  $\Theta$  the mark-up over the marginal costs. The price equation given in (4) can be seen as a reduced form, since the determinants of the mark-up are not derived from a structural oligopoly model. Instead the mark-up is specified as a function of some exogenous variables. The empirical specification is discussed in Section 3.

On the basis of equations (2) and (3) the short-run demand elasticities can be easily calculated; for example, the own price elasticity of labour demand is given by  $\varepsilon_{L,q_l}$  :

$$\varepsilon_{L,q_l} = \frac{\partial L}{\partial q_l} \frac{q_l}{L} = \frac{\partial^2 CV}{\partial q_l^2} \frac{q_l}{\partial CV / \partial q_l} \quad (5)$$

The elasticities  $\varepsilon_{M,q_m}$ ,  $\varepsilon_{L,q_m}$ ,  $\varepsilon_{M,q_l}$ ,  $\varepsilon_{L,Y}$ ,  $\varepsilon_{M,Y}$ ,  $\varepsilon_{L,K}$  and  $\varepsilon_{M,K}$  are determined in an analogous manner.

## 2.2 Employment Thresholds

For studying the impact of output changes on labour demand, instead of the demand elasticities of labour the concept of the “employment threshold” is frequently used in empirical economic research and in the economic-policy discussion (e.g. Siebert 1999). The employment threshold is meant to indicate the output growth rate at which employment remains constant. If the output growth rate is higher (lower) than the threshold, employment would increase (decrease).

In the empirical literature (see e.g., Klauder 1990, Hof 1994, Weeber 1997) it is generally assumed that the employment threshold may vary over time due to decelerations or accelerations of autonomous technical progress, but is independent of factor prices. Consequently, the employment threshold is determined empirically by a simple regression of the growth rate of employment or the growth rate of labour productivity on the growth rate of output (Okun’s Law or Verdoorn’s Law). The typical empirical result for Germany is that the estimated employment threshold shows a steady decline over the last decades. For instance, Klauder (1990) reports that the employment threshold was about 3.8 per cent during the 1960s and 1.2 per cent during the 1980s. This decline is usually attributed to the productivity slowdown.

The most serious shortcoming of these studies is the complete neglecting of the role factor prices play for the determination of the input mix and hence for the employment threshold. To remove this deficiency we derive the employment threshold from the theoretically well founded labour-demand function (2). For this we take the total differential of the labour demand function with respect to time:

$$\frac{dL}{dt} = \frac{\partial^2 CV}{\partial q_l^2} \frac{dq_l}{dt} + \frac{\partial^2 CV}{\partial q_l \partial K} \frac{dK}{dt} + \frac{\partial^2 CV}{\partial q_l \partial q_m} \frac{dq_m}{dt} + \frac{\partial^2 CV}{\partial q_l \partial Y} \frac{dY}{dt} + \frac{\partial^2 CV}{\partial q_l \partial t} \quad (6)$$

In order to calculate the employment threshold, we set  $\frac{dL}{dt}$  in equation (6) to zero and get after

some manipulations the short-run employment threshold  $w_Y^s$ :

$$w_Y^s = - \left( \varepsilon_{L,q_l} (w_{q_l} - w_{q_m}) + \varepsilon_{L,K} w_K + \varepsilon_{L,t} \right) \varepsilon_{L,Y} \quad (7)$$

where  $w_{(\cdot)}$  represents the growth rate for the respective variable,  $\varepsilon_{L,t} = \frac{\partial^2 CV}{\partial q_l \partial t} \frac{1}{L} = \frac{\partial L}{\partial t} \frac{1}{L}$  is the growth rate of employment due to autonomous technological progress and the other elasticities are defined in equation (5).  $w_Y^s$  indicates the short-run employment threshold, i.e. the output growth necessary so that labour input remains constant at given growth rates of factor prices and the capital stock.

Equation (7) shows that the short-run employment threshold depends on the growth rate of relative factor prices, capital accumulation and technical change. Studies which specify the employment threshold as a constant or as a simple function of time are thus based on theoretically unsatisfactory assumptions. The same is true for Okun type regressions of the bivariate relationship between the change in the unemployment rate and output growth. Since factor prices, labour supply and productivity are intervening variables we should not expect a stable relationship in a simple bivariate relationship between changes in unemployment and output. Blinder (1997) has stated that Okun's Law is "atheoretical, if not indeed antitheoretical".

The short-run factor demand elasticities and the employment threshold do not allow for the long-run substitution possibilities between the variable inputs and the quasi-fixed inputs. The long-run demand elasticities and the long-run employment threshold can also be derived from

the variable cost function. To begin with, we will present the derivation of the long-run own price elasticity of labour demand (for the more general case with more than one quasi-fixed production factor, see Brown and Christensen (1981)).

For the derivation of long-run factor demand, the long-run optimal capital stock is needed as a first step. This can be calculated from the following first-order condition

$$\left. \frac{\partial CV(q_m, q_l, K, Y, t)}{\partial K} \right|_{K^*} + q_k = 0 \quad (8)$$

The condition states that the shadow price of capital (the reduction of variable costs due to an additional unit of capital) must correspond in optimum to its one-period user cost  $q_k$ .

If all inputs are at their long-run equilibrium, the long-run total costs can be described as the sum of the variable costs and the capital costs:

$$CT(K^*) = CV(q_m, q_l, K^*, Y, t) + q_k K^*, \quad (9)$$

whereby  $K^*$  indicates the long-run optimal level of  $K$  which is calculated using equation (8).

The partial derivative of  $CT$  with respect to the wage rate indicates the long-run labour demand function

$$L^* = \frac{\partial CT(K^*)}{\partial q_l} = \frac{\partial CV(K^*)}{\partial q_l} + \left[ \frac{\partial CV(K^*)}{\partial K^*} \frac{\partial K^*}{\partial q_l} + q_k \frac{\partial K^*}{\partial q_l} \right], \quad (10)$$

where the expression in brackets is zero according to equation (8). The difference between the short-run (2) and the long-run labour demand (10) is that in the first case the historically given value  $K$  is used and in the second case the long-run optimal value  $K^*$ .

The long-run reaction of factor demand on exogenous changes of factor prices is obtained from the second partial derivative of  $CT$ :

$$\frac{\partial L^*}{\partial q_l} = \frac{\partial^2 CT}{\partial q_l^2} = \frac{\partial^2 CV}{\partial q_l \partial q_l} + \frac{\partial^2 CV}{\partial q_l \partial K^*} \frac{\partial K^*}{\partial q_l} + \frac{\partial K^*}{\partial q_l} \left( \frac{\partial^2 CV}{\partial K^* \partial q_l} + \frac{\partial^2 CV}{\partial K^{*2}} \frac{\partial K^*}{\partial q_l} \right) + \left[ \frac{\partial CV}{\partial K^*} \frac{\partial^2 K^*}{\partial q_l^2} + q_k \frac{\partial^2 K^*}{\partial q_l^2} \right],$$

where again, following from equation (8), the expression in brackets is zero. Equation (11) also requires a calculation of the expression  $\partial K^* / \partial q_l$ . For many functional forms of the cost function, especially also for the translog specification, there is no analytical solution of equation (8) for  $K^*$  and thus also no analytical solution for the required partial derivative in (11).

With the application of the total differential and the implicit function theorem on equation (8), the necessary partial derivative is received taking into consideration

$$dY = dt = dq_k = dq_m = 0, :$$

$$\frac{\partial K^*}{\partial q_l} = - \frac{\frac{\partial^2 CV}{\partial K^* \partial q_l}}{\frac{\partial^2 CV}{\partial K^{*2}}} \quad (12)$$

Thus also the expression in parentheses in equation (11) is zero. With equation (12) the calculation of equation (11) for any functional form of the cost function is possible. All derivatives are calculated at the point  $K = K^*$ . Thus, from (11) we have the following expression of the long-run own-price elasticity of labour demand

$$\eta_{L,q_l} = \left( \frac{\partial^2 CV}{\partial q_l^2} + \frac{\partial^2 CV}{\partial q_l \partial K^*} \frac{\partial K^*}{\partial q_l} \right) \frac{q_l}{\partial CT / \partial q_l}, \quad (13)$$

whereby  $\partial K^* / \partial q_l$  is substituted from equation (12). The other long-run elasticities for L and M with respect to the factor prices  $q_m$ ,  $q_l$  and  $q_k$  are calculated analogously. The long-run elasticities of capital demand are obtained directly by applying the implicit function theorem on the first-order conditions for the optimal capital use in equation (8).

Analogously to (7) by the total derivative of the long-run labour demand function we obtain the long-run employment threshold  $w_Y^l$ :

$$w_Y^l = - \left( \eta_{L,q_l} (w_{q_l} - w_{q_m}) + \eta_{L,q_k} (w_{q_k} - w_{q_m}) + \eta_{L,t} \right) / \eta_{L,Y} \quad (14)$$

with the long-run elasticities from (13) as well as  $\eta_{L,t} = \frac{\partial^2 CT(K^*)}{\partial q_l \partial t} \frac{1}{L} = \frac{\partial L^*}{\partial t} \cdot \frac{1}{L^*}$ . The long-run employment threshold indicates the necessary output change as a function of factor price changes and technical progress in order that employment remains constant even when taking additional long-run substitution possibilities between the variable and the quasi-fixed inputs into consideration. Equations (7) and (14) allow to calculate the impact of hypothetical factor price changes on the short-run and long-run employment threshold in a straightforward manner.

### 3 Empirical specifications and data

In order to calculate the demand elasticities and employment thresholds discussed in Section 2 empirically, a parametric variable cost function has to be specified. In the following, we will use the translog function, which, for example, is the basis of the studies by Brown/Christensen (1981) and Berndt/Hesse (1986) and which presents a flexible functional form (see e.g. Chambers 1988, Ch.5 or Morrison 1992, Ch.7.2).

For our model with the variable inputs of labour and intermediate products and the quasi-fixed input capital, the variable translog cost function is given by

$$\begin{aligned} \ln CV = & \alpha_0 + \alpha_Y \ln Y + \alpha_l \ln q_l + \alpha_m \ln q_m + \alpha_k \ln K + \alpha_t t + 0.5\gamma_{YY} (\ln Y)^2 + 0.5\gamma_{ll} (\ln q_l)^2 \\ & + 0.5\gamma_{mm} (\ln q_m)^2 + 0.5\gamma_{kk} (\ln K)^2 + 0.5\gamma_{tt} t^2 + \gamma_{Yl} \ln Y \ln q_l + \gamma_{Ym} \ln Y \ln q_m + \gamma_{Yk} \ln Y \ln K \\ & + \gamma_{Yt} t \ln Y + \gamma_{lm} \ln q_l \ln q_m + \gamma_{lk} \ln q_l \ln K + \gamma_{lt} t \ln q_l + \gamma_{mk} \ln q_m \ln K + \gamma_{mt} t \ln q_m + \gamma_{kt} t \ln K \end{aligned} \quad (15)$$

The necessary condition of linear homogeneity in the input prices implies (see e.g. Berndt/Hesse 1986)

$$\begin{aligned} \alpha_l + \alpha_m &= 1 & \gamma_{lt} + \gamma_{mt} &= 0 \\ \gamma_{Yl} + \gamma_{Ym} &= 0 & \gamma_{ll} + \gamma_{lm} &= 0 \\ \gamma_{lk} + \gamma_{mk} &= 0 & \gamma_{lm} + \gamma_{mm} &= 0 \end{aligned} \quad (16)$$



In order that the underlying dual production function displays constant return to scale, the following necessary and sufficient conditions must be fulfilled (see Brown/Christensen 1981)

$$\begin{aligned}
 \alpha_Y + \alpha_k = 1 & \quad \text{a)} & \quad \gamma_{Ym} + \gamma_{mk} = 0 & \quad \text{d)} \\
 \gamma_{YY} + \gamma_{Yk} = 0 & \quad \text{b)} & \quad \gamma_{Yk} + \gamma_{kk} = 0 & \quad \text{e)} \\
 \gamma_{Yl} + \gamma_{lk} = 0 & \quad \text{c)} & \quad \gamma_{Yt} + \gamma_{kt} = 0 & \quad \text{f)}
 \end{aligned} \tag{17}$$

In using the translog function, the factor demand functions are usually estimated in the form of cost shares (e.g. for the input labour:  $L^* q_l / CV = \partial \ln CV / \partial \ln q_l$ ) together with the cost function  $CV$ . In this study we use the input coefficients as the dependent variables:

$$\frac{L}{Y} = \frac{\partial CV}{\partial q_l} \frac{1}{Y} = \frac{\partial \ln CV}{\partial \ln q_l} \frac{CV}{q_l} \frac{1}{Y} \tag{18}$$

$$\frac{M}{Y} = \frac{\partial CV}{\partial q_m} \frac{1}{Y} = \frac{\partial \ln CV}{\partial \ln q_m} \frac{CV}{q_m} \frac{1}{Y} \tag{19}$$

For the empirical implementation of the price function we assume that the output price is determined by means of a variable mark-up over the short-run marginal costs. The main discussion in the literature is whether the price mark-up rate exhibits a procyclical or a countercyclical behaviour (see e.g. Rotemberg/Woodford 1991). In this paper we assume that the mark-up rate may depend on the growth rate of output.

For this reason we specify the price equation as

$$P = (\beta_0 + \beta_1 w_Y) \frac{\partial CV}{\partial Y} \tag{20}$$

with  $w_Y$  as the growth rate of real output.

Our investigation is based on the West German manufacturing sector for the period from 1968 to 1995. Where no other reference is given, all data are from National Accounts. As measures for real output and intermediate product we use the productions value and intermediate input in 1991 prices. The implicit price deflators serve as prices for output and intermediate input. The

total labour volume is constructed by multiplying the actual annual working hours per worker (source: Görzig et al. 1997) with the number of all employed persons.<sup>1</sup> The nominal costs of employees correspond to income from dependent employment. If this amount is divided by the annual labour volume of employees, we obtain the nominal hourly wage rate.

According to the concept of National Accounts, the purchases of new capital goods are recorded for the purchaser (owner concept). Since leasing and other forms of hiring of capital goods have become more widespread in the business sector since the 1970s, we employ the user concept for the production factor capital. The nominal user costs of capital and the gross capital stock in 1991 prices are based on calculations of the ifo Institute (for the methodology, see *Gerstenberger/Heinze/Hummel/Vogler-Ludwig* 1989). For the gross capital stock, values at the beginning of the year are used.

## 4 Results

### 4.1 Short- and long-run factor demand

The system to be estimated consists of the cost function (15), the short-run input demand functions (18), (19) and the price equation (20). Since the cost function in (15) contains a lot of parameters to be estimated and we only have observations for 28 years, the linear homogeneity of the variable costs functions in the factor prices (equation (16)) were imposed on the estimates. On the other hand, we do not force constant returns to scale onto the estimates in order not to unnecessarily restrict the technology from the very outset.

The model was estimated with the non-linear, three-stage least squares method. A potential problem is which of the explanatory variables can be treated as weakly exogenous in the sense that they are uncorrelated with the contemporary disturbances in our equation system. Since cost or price shocks induce probably a reaction of output demand within the observation period of a year we treat output as an endogenous variable. The case is not so clear-cut for the input prices.

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<sup>1</sup> It is assumed that the annual hours worked by dependent employees and by self-employed as well as by helping family members do not differ.

The wage rate is determined to a great extent in wage negotiations at the beginning of a year so that an unexpected shock in factor demand should have only a modest effect on the current wage rate. The variability of the price of intermediate products is dominated by changes of raw material prices which are largely determined in the world market.

For the estimation of the equation system, we use three different sets of instrumental variables in order to cope with the problem of a potential endogeneity of the right-hand side variables. In all sets are included the trend terms, the capital stock and the lagged values of the endogenous variables, of output and of factor prices. In set I, for which the estimation results are presented below, the current values of the factor prices are also included. In set II, we exclude the current wage rate which means that this variable is allowed to be endogenous. All estimation results remain essentially the same. When we use set III, in which we exclude in addition the current price of intermediate inputs, some estimated parameters change slightly but the calculated values for the elasticities and the employment threshold imply the same qualitative conclusions as the results presented here. We prefer set I since the lagged value of the material price is a weak predictor for the current price and is therefore a bad instrument.

After estimating the unrestricted system, we tested for constant returns to scale. To do this the restrictions in (17) were examined both individually and simultaneously with the Wald test. The hypothesis of constant returns to scale was rejected (Wald test statistics = 177 at 5 degrees of freedom).<sup>2</sup> It became apparent, however, that restriction (17f) is completely insignificant. Since in addition also the individual parameters of this restriction proved to be insignificant (t-values: -0.56 and -0.56), these parameters were set equal to zero for the final estimation in order to achieve the most efficient specification.

The cost function must satisfy certain regularity conditions (see Section 2) in order to be able to describe the technology and firms' cost-minimising behaviour. They are all fulfilled here since the cost function in our estimates for each year proved to be increasing and concave in the factor prices and decreasing and convex in the quasi-fixed factor capital.

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<sup>2</sup> On the basis of the restrictions in (16), we only receive five linearly independent restrictions in (17), since in the case of linear homogeneity in input prices, the restrictions (17c) and (17d) are identical.

The estimated parameter and the accompanying t-values as well as the summary statistics are displayed in Tables 1 and 2. Nearly all parameters are highly significant. Thus, simple cost or production functions like Cobb-Douglas or CES functions are not suitable for depicting all economically relevant aspects of technology. On the whole, the adjustment of the model to the data is very good (see Table 2): In every equation  $R^2$  exceeds 98 percent, and there are only weak signs of auto-correlation of the residuals. The Durbin-Watson coefficient indicates only for the labour demand equation a possibly significant autocorrelation. This is confirmed by an examination of the correlograms. All autocorrelations are within the two-standard-error band with the exception of lag one for the residuals of the labour demand equation, where the autocorrelation coefficient has a t-value of 2.1. Autocorrelation may be a sign of mis-specification but may also be attributable to the use of yearly data. As is well known (see Weiss 1984), temporal aggregation leads to a moving average process for the residuals. If this is the main reason for the observed autocorrelation, our specification is tenable but the lagged values of the dependent variables would not be valid instruments. Using set I without the lagged dependent variables as instruments leaves the empirical conclusions largely unaffected. So we are confident that the results we derive in the following are based on a correctly specified model. In addition, a comparison of the t-values in columns three (Gauss-Newton-method) and four (White method) of Table 1 indicates that no significant heteroscedasticity problems occur. Finally, Table 2 displays some diagnostics for the residuals of the estimated equations. The CUSUM-stastic is defined as the maximum of the ratio of the accumulated standardized residuals to the upper 95 % confidence bound. If this test statistic is greater than one, we have an indication for model misspecification (for a discussion of the CUSUM-test see Harvey 1990). ARCH is a test for first-order conditional heteroscedasticity. Jarque-Bera is a Lagrange Multiplier of the residual's skewness and kurtosis. On the whole, there is no indication for model misspecification.

The highly significant parameters  $\beta_0$  and  $\beta_1$  describe the price setting behaviour of the firms. The estimates show that rising marginal costs lead enterprises to increase their prices. In case of perfect competition,  $\beta_0$  must be one, since this parameter measures the average markup over the marginal costs. This null hypothesis, however, is clearly rejected (Wald test statistic = 7.78 with one degree of freedom). Parameter  $\beta_1$  measures the cyclical variability of the markup. The parameter is positive which means that the mark-ups display a procyclical behaviour.

For 1990, the short- and long-run elasticities of factor demand are given in Table 3. The short-run elasticity of labour demand with respect to the wage rate is -0.17. The long-run absolute elasticity of labour with respect to the wage rate is approximately double that of the short-run. Since the short-run cross price elasticities of the variable inputs do not differ from the long-run, this effect must take place by means of an adjustment of the quasi-fixed factor capital to wage changes. As seen in Table 3, the long-run elasticity of capital demand with respect to the wage rate is approximately 0.32. Thus, the long-run substitution of labour by capital makes a major contribution to the higher long-run elasticity of labour demand. Wage increases thus lead in the long run to a stronger decline in employment than the short-run reaction of labour demand indicates.

**Table 1: Parameter estimations**

Parameter <sup>1</sup>	Estimation	t-value	t-value <sup>2</sup> (White)
$\alpha_0$	4.9904	1.42	1.29
$\alpha_Y$	1.3824	8.34	7.47
$\alpha_l$	1.3890	18.05	18.64
$\alpha_k$	-1.5507	-1.54	-1.40
$\alpha_t$	-0.0144	-8.53	-8.51
$\gamma_{tt}$	0.0004	4.90	4.65
$\gamma_{YY}$	0.1201	3.51	4.55
$\gamma_{ll}$	0.1566	21.87	27.29
$\gamma_{kk}$	0.3615	2.38	2.21
$\gamma_{Yl}$	-0.0047	-0.43	-0.42
$\gamma_{Yk}$	-0.1715	-4.06	-4.41
$\gamma_{mk}$	0.1311	9.81	7.86
$\beta_0$	0.0028	11.12	8.91
$\beta_1$	1.0829	43.20	36.43
	0.2350	5.08	5.19

<sup>1)</sup> For parameters  $\alpha_m$ ,  $\gamma_{lm}$ ,  $\gamma_{mm}$ ,  $\gamma_{Ym}$ ,  $\gamma_{lk}$ ,  $\gamma_{lt}$  see the restrictions in (16).

<sup>2)</sup> Calculated with the help of a heteroscedasticity-consistent variance/covariance matrix.

**Table 2: Summary statistics**

Equation	$R^2$	Durbin-Watson	ARCH	CUSUM	Jarque-Bera
<i>CV</i>	0.9997	1.2695	0.147	0.450	0.127
<i>L/Y</i>	0.9965	1.1162	0.618	0.457	1.082
<i>M/Y</i>	0.9850	1.5771	1.387	0.547	1.410
<i>P</i>	0.9967	1.4471	0.444	0.351	3.041
Critical values (5 %)	---	---	3.84	1	5.99

In contrast, the long-run elasticities of demand for intermediate input hardly differs from that of the short run, which is not surprising given the low cross-price elasticities between intermediate input and capital. In the long run, all three production factors are Allen substitutes, with a very low substitutional relationship between intermediate input and capital. All-in-all, intermediate input shows the least price elasticity. Similar results were obtained by Falk and Koebel (1997) for the West German manufacturing industry.

The short-run elasticities of the variable inputs with respect to output is in both cases approximately one. In the short-run we thus have a homothetic technology in the variable production factors, since in the case of output change, *ceteris paribus*, the factor input ratio does not change. In the long run, however, we cannot assume a homothetic technology. The long-run elasticity of labour demand with respect to output falls to approximately 0.62, but the elasticity of intermediate input demand hardly changes. Also the elasticity of capital demand with respect to output is considerably below one. The long-run output elasticities thus show that there are increasing returns to scale in the manufacturing industry.

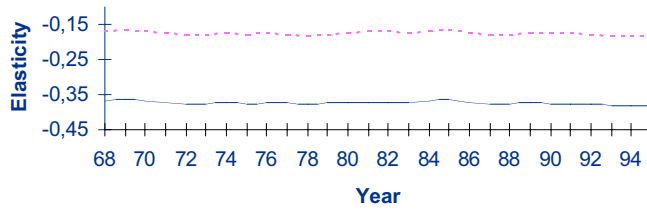
**Table 3: Elasticities of factor demand (1990)**

	Short run			Long run		
	Labour	Intermediate input	Capital	Labour	Intermediate input	Capital
$q_l$	-0.174	0.073	-	-0.375	0.071	0.318
$q_m$	0.174	-0.073	-	0.170	-0.072	0.005
$q_k$	-	-	-	0.205	0.001	-0.323
$Y$	1.020	1.043	-	0.622	1.040	0.627
$K$	-0.635	-0.003	1	—	—	—

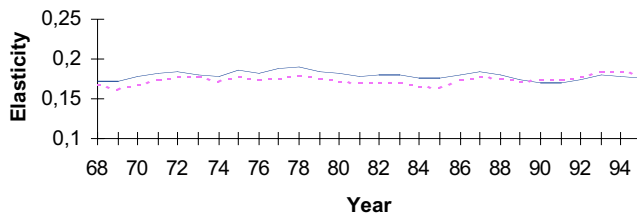
In Figure 1 the time path of all estimated demand-elasticities is displayed. With the exception of the elasticity with respect to time the estimated short- and long-run elasticities proved to be very stable for the whole period. The negative effect of “time” on labour demand declined by half over the estimation period. This is attributable to the productivity slowdown, since at a lower growth rate of the total factor productivity, the reduction in labour demand due to autonomous technical progress gets smaller.

**Fig. 1: Elasticities of labour demand with respect to wages, intermediate input prices, output, technical progress and capital stock**

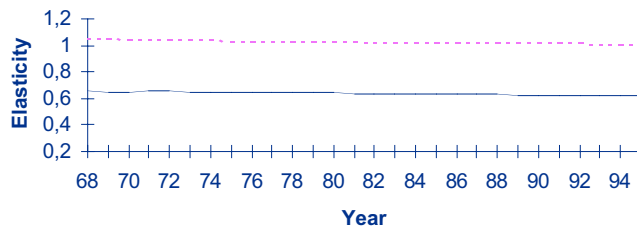
**Wages**



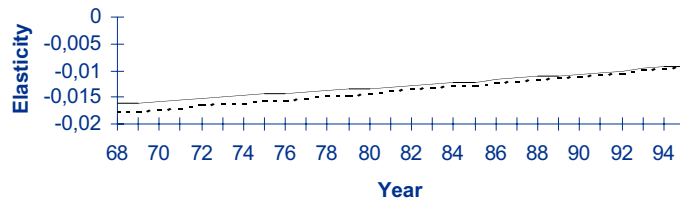
**Price of intermediate products**



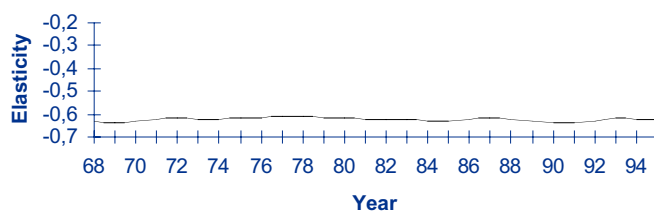
**Output**



**Technical progress**



**Stock of capital**



-----short run    \_\_\_\_\_long run



## 4.2 Employment thresholds

In Section 2 it was shown that the employment threshold and the input-elasticities are closely related. Based on the estimated parameters of our empirical model we have calculated the short-run employment threshold  $w_Y^s$  using equation (7) and the long-run employment threshold  $w_Y^l$  from equation (14) for each year in our sample at the realised historical growth rates of factor prices and the capital stock (Figure 2). Whereas  $w_Y^s$  has a direct empirical interpretation,  $w_Y^l$  shows the hypothetical employment threshold which would hold if the capital stock were always on the long-run equilibrium value.

Employment thresholds are not observed but rather estimated variables. In order to assess the quality of the estimation, we calculated the standard error of the employment threshold as a function of the standard errors of the estimated model parameters. Figure 3 shows the results for the short-run employment threshold. The calculated standard errors are sufficiently small to be confident that the variation of the estimated employment threshold over time is not dominated by parameter uncertainty.

If the elasticities used in equation (7) and (14) are constant over time, these equations can be interpreted as regression equations, at which the estimated coefficients reflect the effects of factor prices and the capital stock on the employment threshold. The estimated coefficient for the intercept shows the influence of technical progress on the employment threshold. If the elasticities are not constant over time and exhibit high volatility it would not be reasonable to run a regression with constant coefficients. As Figure 1 shows, the elasticities are not really constant but very stable. The only exception is the effect of technical progress which can however be modelled as a linear function of time.

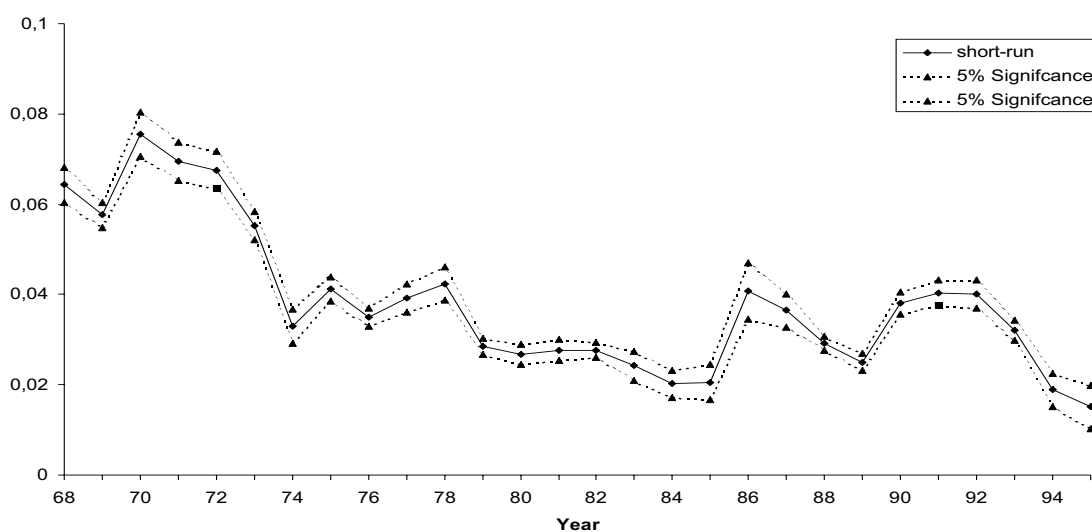
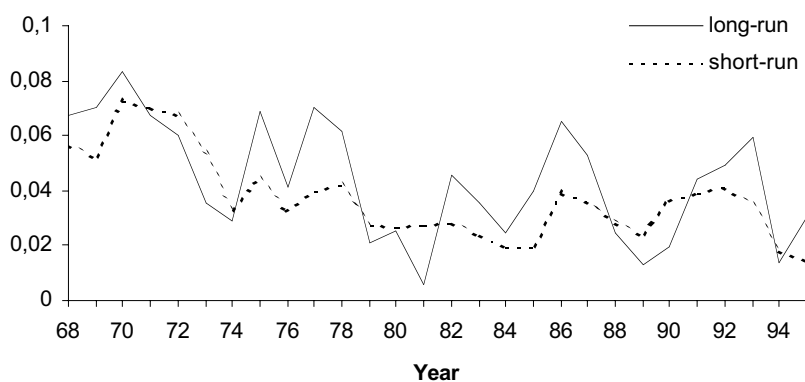
**Fig. 2: Short-run and long-run employment threshold****Figure 3: Short-run employment threshold and 95 % confidence bounds**

Table 4 shows two versions of OLS estimates for the short-run employment threshold. The estimates in version I take as explanatory variables only the growth rates of the relative factor prices and the capital stock, in addition to a constant term. The very low Durbin-Watson Test statistic points to a potential miss-specification. For this reason and for capturing the decreasing growth rate of total factor productivity, we have additionally included a time trend in Version II.

This improved the Durbin-Watson statistic considerably and all variables are highly significant. The four variables in the regression suffice to explain the variation of  $w_Y^s$  almost completely. The corrected  $R^2$  amounts to nearly 99 percent. This allows to depict the relationship between the employment threshold and its determinants as a linear one. The influence of technical progress on the employment threshold has declined over time. In the mid 1990's, the employment threshold is about 1 percent lower than at the end of the 1960's. This reflects the effect of the productivity slowdown. If the wage rate increases by 1 percent or if the intermediate product prices fall by 1 percent, output must rise by an additional 0.17 percentage points so that employment remains constant. This only takes into account the short-run substitution possibilities between the labour and the intermediate product input.

**Table 4: OLS estimates for short-run employment threshold  $w_Y^s$**

Parameter	Version I		Version II	
	Estimate	t-value	Estimate	t-value
Constant	0.011	10.02	0.020	12.69
$\Delta \ln(q_1/q_m)$	0.173	11.13	0.174	17.75
$\Delta \ln K$	0.704	19.05	0.579	18.91
Trend	--	--	-0.0004	-6.24
$\bar{R}^2$	0.968		0.987	
Durbin-Watson	0.684		1.179	

In Table 5, the results for the analogous regression for the long-run employment threshold are presented. Explanatory variables are the growth rates of the relative input prices and in Version II a trend term.

The coefficient on the growth rate of the wage implies that a one percent wage increase requires, *ceteris paribus*, in the long run an output increase of approximately 0.6 percent to leave labour input unchanged. Therefore, in the long run, the output growth needs to be three times as high as in the short-run. This is for two reasons. Firstly, the long-run labour-demand elasticity is approximately double that of the short-run labour-demand elasticity. Secondly, labour demand reacts less to output increases in the long run than in the short run so that a high output growth is necessary in order to stabilise employment.

**Table 5: OLS estimates for long-run employment threshold  $w_y^l$**

Parameter	Version I		Version II	
	Estimate	t-value	Estimate	t-value
Constant	0.019	28.21	0.026	53.85
$\Delta \ln(q_l/q_m)$	0.613	48.96	0.586	146.18
$\Delta \ln(q_k/q_m)$	-0.333	-26.22	-0.311	-78.73
Trend	--	--	-0.0003	-16.42
$\bar{R}^2$	0.989		0.999	
Durbin-Watson	0.420		1.71	

## 5 Summary and conclusions

In this study, we have analysed the labour demand of the manufacturing industry on the basis of a variable translog cost function. We found that there are significant and important substitution effects and that thus wage increases lead to a reduction of labour demand, at a given output level. The short-run and long-run elasticities of labour demand with respect to the wage rate amounts to  $-0.17$  and  $-0.37$ , respectively. This substitution process has strong implications for the employment threshold which denotes the growth rate of output necessary for holding employment constant. The employment threshold depends on the growth rates of input prices and the rate of autonomous technical progress. The empirical results show that an increase in the wage rate by 1 per cent – leaving all other input prices constant – requires an increase in output by 0.17 per cent in the short-run and by about 0.6 per cent in the long-run in order to keep employment constant.

These findings shed some light on the ongoing discussion concerning the possibility to raise employment by means of an expansionary wage policy. Due to the substitution effect the proportional increase in the nominal wage bill will be lower than the increase in the wage rate and due to the induced increase in the output price, the increase in the real wage bill will be even lower. In addition, price increases reduce the international competitiveness of domestic industry, which causes a reduction of exports and an increase in import demand. If the firms were not able to transfer the cost increases to their prices, their real profits would decline and thus also the consumption and investment demand that is dependent on profits. For all these reasons it seems highly unlikely to us that a one-percent wage increase would raise overall demand in the long run by the same amount as the employment threshold (0.6 percent) increases. Wage increases thus induce a reduction in employment.

As this discussion shows, the concept of “employment threshold” may be a useful tool for analysing some aspects of the employment problem. But this is only the case if the concept is correctly applied. The widespread belief that employment thresholds are something like a structural characteristic is very misleading. The employment threshold depends crucially on the time path of wages and the prices of intermediate products as well as on capital accumulation in the short run and user cost of capital in the long run. It evolves in a complicated way over time when

some of its determinants are changed. There is no simple coefficient which can capture all these aspects.

There is one additional complication when we are interested not only in total labour volume, which is the object of this study, but also in the number of employed persons (see Buscher et al., 2000). This variable is typically thought to be a quasi-fixed input since hiring and firing is associated with severe restrictions and high adjustment costs. The employment threshold for the number of workers may be an even more complicated dynamic function of the determinants of employment. The analysis of this problem within a dynamic cost-minimising framework is left for future research.

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